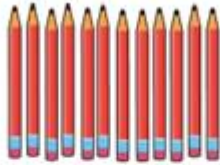


Step 1: To understand that objects can be grouped in to equal and unequal groups

'There are some pencils.'



'The pencils have been grouped.'

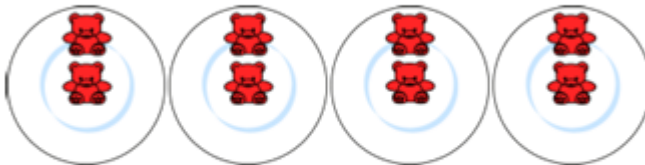
'There are some footballs.'



'The footballs have been grouped.'

Step 2: To know that when we describe equally grouped objects both the number of groups and the size of groups must be defined.

- 'How many equal groups are there?'
- 'How many bears are there in each group?'



- 'There are four equal groups of bears.'
- 'There are two bears in each group.'
- 'There are four groups of two.'

Step 3: To know that equal groups can be represented with repeated addition.



2	2	2	2
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$$2 + 2 + 2 + 2$$

- 'There are four groups of two.'
- 'There are two and two and two and two.'
- 'We can write this as two plus two plus two plus two.'

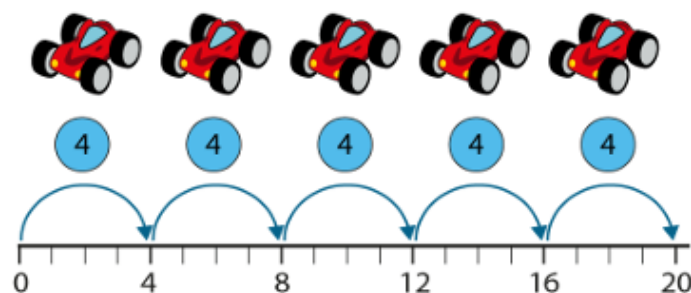
Step 4: To know that equal groups can be represented with multiplication.

'A snail moved 10 cm in the morning, another 10 cm in the afternoon, a further 10 cm in the evening and 10 cm overnight.'

	
'There are four groups of ten.'	
'We can write this as <u>ten plus ten plus ten plus ten</u>.' $10 + 10 + 10 + 10$	'We can also write this as four times ten.' 4×10

Step 5: Use of repeated addition on a numberline.

'How many wheels are there? Count in groups of four.'

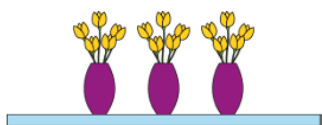


- 'Four, eight, twelve, sixteen, twenty. There are twenty wheels.'
 - 'There are five groups of four; there are twenty altogether.'
 - 'There are four, five times; there are twenty altogether.'
- $$5 \times 4 = 20 \qquad 4 \times 5 = 20$$
- 'Five is a factor.'
 - 'Four is a factor.'
 - 'The product of five and four is twenty.'
 - 'Twenty is the product of five and four.'

Step 6: To understand that multiplication is commutative.

Commutativity of multiplication – ‘one interpretation, two equations’:

‘Write two multiplication equations to represent this picture.’



$$3 \times 5 = 15$$

• ‘Three groups of five are equal to fifteen’

$$5 \times 3 = 15$$

• ‘Five, three times is equal to fifteen.’

• ‘Three groups of five can be written as three times five, or as five times three.’

• ‘What’s the same?’

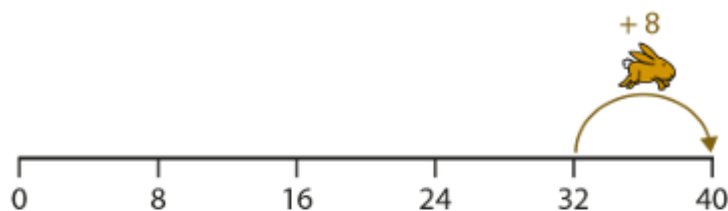
• ‘In both equations “3” and “5” are factors and “15” is the product.’

• ‘What’s different?’

• ‘The factors are written in a different order.’

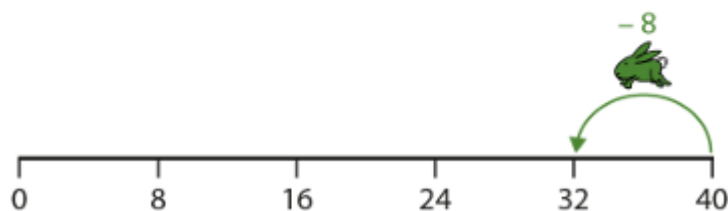
Step 7: To understand that multiplication is distributive and we can use known facts to be efficient.

Adding one multiple:



$$5 \times 8 = 4 \times 8 + 8$$

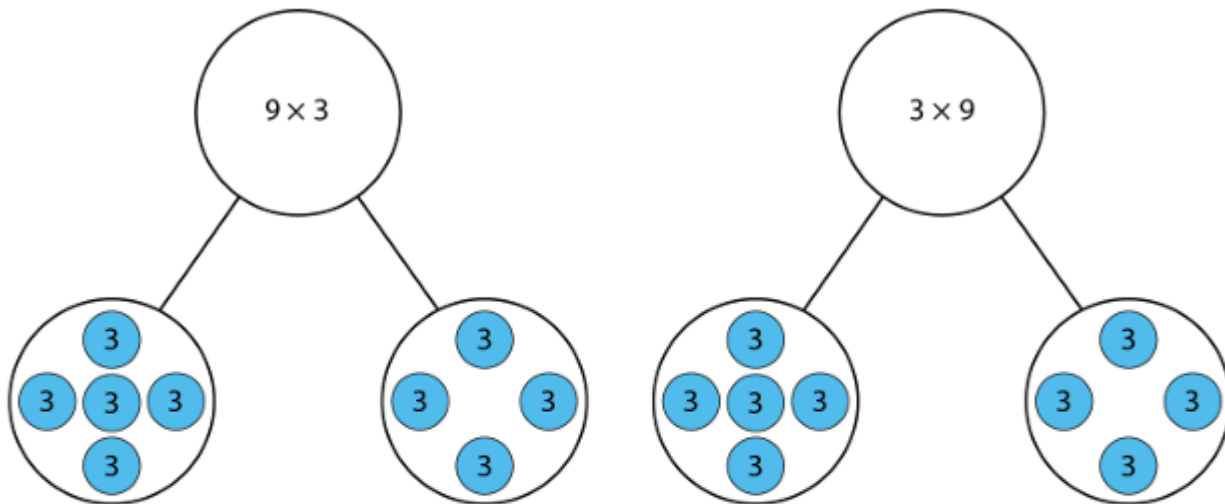
Subtracting one multiple:



$$4 \times 8 = 5 \times 8 - 8$$

Step 8: To explore with varying the order of the factors.

Varying the order of the factors:



$$9 = 5 + 4$$

$$\begin{aligned} 3 \times 9 &= 3 \times 5 + 3 \times 4 \\ &= 15 + 12 \\ &= 27 \end{aligned}$$

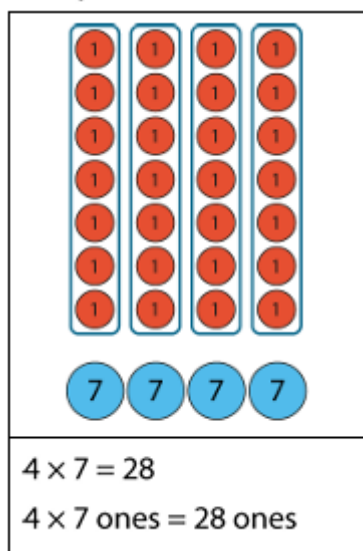
'Nine is equal to five plus four, so three times nine is equal to three times five plus three times four.'

$$5 = 9 - 4$$

$$\begin{aligned} 3 \times 5 &= 3 \times 9 - 3 \times 4 \\ &= 27 - 12 \\ &= 15 \end{aligned}$$

'Five is equal to nine minus four, so three times five is equal to three times nine minus three times four.'

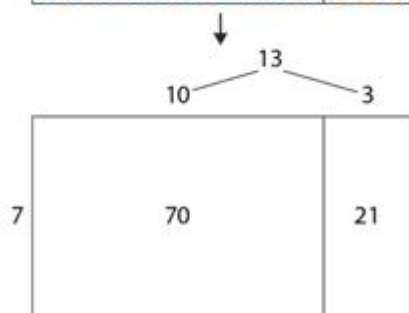
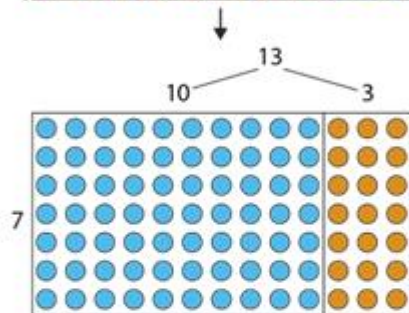
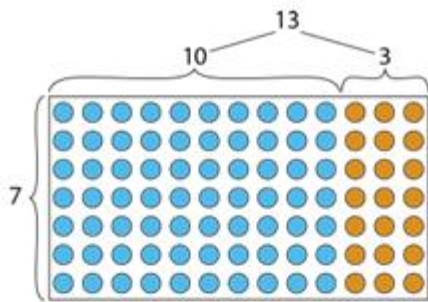
Step 9: To use arrays to explore factors and products.



$$4 \times 7 = 28$$

$$4 \times 7 \text{ ones} = 28 \text{ ones}$$

Step 10: To partition to find partial products and recombine in a grid method.



$$\begin{aligned}
 13 \times 7 &= 10 \times 7 + 3 \times 7 \\
 &= 70 + 21 \\
 &= 91
 \end{aligned}
 \qquad
 \begin{aligned}
 7 \times 13 &= 7 \times 10 + 7 \times 3 \\
 &= 70 + 21 \\
 &= 91
 \end{aligned}$$

'One pack of biscuits costs 84 p. How much do six packs cost?'

Example working 1:

$$\begin{array}{r}
 84 \\
 \swarrow \searrow \\
 80 \quad 4
 \end{array}
 \times 6 = 504$$

$$80 \times 6 = 480$$

$$4 \times 6 = 24$$

$$480 + 24 = 504$$

'Six packs of biscuits cost 504 p.'

Step 11: To use formal multiplication (expanded method)

Multiplication algorithm – expanded layout:

Step 1 – write the factors:

	10s	1s
	3	4
×		2

Step 2 – multiply the single-digit number by the ones:

	10s	1s
	3	4
×		2
		8

$2 \times 4 \text{ ones} = 8 \text{ ones}$

Step 3 – multiply the single-digit number by the tens:

	10s	1s
	3	4
×		2
		8
	6	0

$2 \times 4 \text{ ones} = 8 \text{ ones}$
 $2 \times 3 \text{ tens} = 6 \text{ tens}$

Step 4 – add the partial products:

	10s	1s
	3	4
×		2
		8
	6	0
	6	8

$2 \times 4 \text{ ones} = 8 \text{ ones}$
 $2 \times 3 \text{ tens} = 6 \text{ tens}$

Step 12: To use formal multiplication (compact method) and include regrouping

Example 1 – compact layout *with* place-value headings:

	10s	1s
	3	2
×		3
	9	6

- $3 \times 2 \text{ ones} = 6 \text{ ones}$
'Write "6" in the ones column.'
- $3 \times 3 \text{ tens} = 9 \text{ tens}$
'Write "9" in the tens column.'

Example 2 – compact layout *without* place-value headings:

	2	1
×		4
	8	4

- $4 \times 1 \text{ one} = 4 \text{ ones}$
'Write "4" in the ones column.'
- $4 \times 2 \text{ tens} = 8 \text{ tens}$
'Write "8" in the tens column.'

Example 1 – compact layout *with* place-value headings:

Step 1 – write the factors:

	10s	1s
	2	4
×		3

Step 2 – multiply the single-digit number by the ones and regroup:

	10s	1s
	2	4
×		3
		2
	1	

$3 \times 4 \text{ ones} = 12 \text{ ones}$
 $= 1 \text{ ten} + 2 \text{ ones}$

'Write "1" below the tens column and "2" in the ones column.'

Step 3 – multiply the single-digit number by the tens and add the tens from regrouping:

	10s	1s
	2	4
×		3
	7	2
	1	

$3 \times 2 \text{ tens} = 6 \text{ tens}$

$6 \text{ tens} + 1 \text{ ten} = 7 \text{ tens}$
'Write "7" in the tens column.'

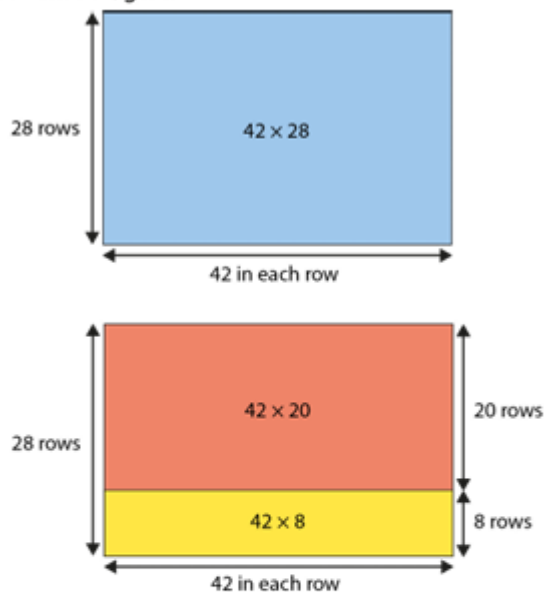
Step 13: To use formal compact multiplication compact method with larger numbers x1 digit

Multiplication algorithm – compact layout:

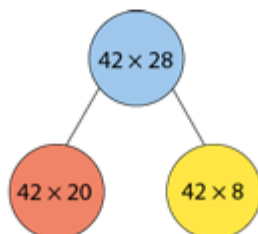
	3	6	7
×			4
	1	4	6
	2	2	

Step 14: To use known facts and methods of partitioning to multiply 2x2 digit numbers

Area model/grid:



Part-part-whole model:



Multiplication algorithm – compact layout:

$$\begin{array}{r} 31 \\ \times 24 \\ \hline 124 \\ 620 \\ \hline 744 \end{array}$$

Multiplication algorithm – compact layout:

$$\begin{array}{r} 312 \\ \times 28 \\ \hline 2496 \\ 6240 \\ \hline 8736 \\ 1 \end{array}$$

Step 15: To use known facts to apply and link to multiplying decimals.

$3 \times 4 = 12$	$3 \times 0.4 = 1.2$	$3 \times 0.04 = 0.12$
$3 \times 4 \text{ ones} = 12 \text{ ones}$	$3 \times 4 \text{ tenths} = 12 \text{ tenths}$	$3 \times 4 \text{ hundredths} = 12 \text{ hundredths}$